Performance assessment of a dynamic current allocator for the JET eXtreme Shape Controller

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\begin{abstract}
This paper reports on a recently proposed dynamic allocation technique that can be effectively adopted to handle the current saturations of the Poloidal Field coils with the eXtreme Shape Controller. The proposed approach allows to automatically relax the plasma shape regulation when the reference shape requires current levels out of the available ranges, finding in real-time an optimal trade-off between shape control precision and currents saturation avoidance. In this paper the results attained during preliminary analysis are presented, showing the advantage arising from the use of the dynamic allocator, versus the bare use of the eXtreme Shape Controller.
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1. Introduction

The need for achieving better performance in present and future tokamak devices is pushing plasma control to gain increasing importance in tokamak engineering. High performance in tokamaks is achieved by plasmas with elongated poloidal cross-section. Strong motivations to improve plasma control are the need to maximize the plasma volume within the available space, and the heat flux control in the divertor region. In particular, the ability to control the plasma shape with an accuracy of a few centimeters is an essential feature of any plasma position and shape control system. The eXtreme Shape Controller (XSC,\textsuperscript{[1, Ch. 9]}) allows to accurately control highly elongated plasmas at the JET tokamak\textsuperscript{[2]} by driving the current in the Poloidal Field (PF) coils. The XSC enables high accuracy control of the overall plasma boundary, specified in terms of a given number of plasma shape descriptors, i.e., gaps, strike-points and x-point. In its present implementation, the XSC does not handle current saturations in the PF coils. Indeed, each operating scenario is carefully designed\textsuperscript{[3]} in order to avoid PF currents saturation in the presence of the envisaged disturbances (i.e., plasma current, poloidal beta and internal inductance variations). A dynamic coil Current Allocator (CA) based on the technique originally proposed in\textsuperscript{[4]} has been recently proposed to manage current limit avoidance with the XSC\textsuperscript{[5]}. The CA exploits the redundancy of the PF coil system to obtain “almost the same plasma reference shape” with different PF currents combinations. Hence, in the presence of disturbances, it aims at avoiding the current saturations by “relaxing” the plasma shape constraints. Furthermore, the CA guarantees an optimal trade-off, at the steady-state, between shape loss and distance of the coil currents from their saturation limits. In this paper a different cost function is used, with respect to the one proposed in\textsuperscript{[5]}, which penalizes the tracking error instead of the deviation from the unallocated output. Also more stress is put on closed-loop simulations of real scenarios in which the CA can actually improve the XSC performances.

The paper is structured as follows: in Section 2 some theoretical results are briefly recalled and the allocation scheme is described, in Section 3 two test cases are presented in order to show the effectiveness of the proposed approach.

2. Theoretical framework

The proposed CA is based on the dynamic input allocation scheme first proposed in\textsuperscript{[4]} for input redundant systems, then generalized to non redundant ones in\textsuperscript{[5]}. In the former case, thanks to the redundancy in the control inputs to the plant, there exist, at least at the steady state, infinitely many selections of these inputs that give the same desired values at the plant output: the allocator makes the control system converge to the optimal input selection, according to some cost function. In the latter case there does not exist in general an input selection that gives the desired output. Indeed, the XSC control algorithm minimizes a quadratic cost func-

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\textsuperscript{1} See Appendix of F. Romanelli et al., IAEA Fusion Energy Conference 2008 (Proc. 22nd Int. Conf., Geneva, Switzerland).
dotted trace is the simulated shape. The shape reference is the dashed trace, while the limits without degrading too much the plasma shape, the allocation combination is not feasible. In order to keep the currents within their of the actuators, it could happen that the requested current com-
tion for the plasma shape error in order to obtain at the steady state the output that best approximates the desired one. However, since the XSC algorithm does not take into account the current limits of the actuators, it could happen that the requested current combination is not feasible. In order to keep the currents within their limits without degrading too much the plasma shape, the allocation scheme proposed in [5] finds an optimal trade-off between these two objectives, specified in terms of an adequate cost function.

Consider the plant described by the linear time-invariant system:

\[ \dot{x} = Ax + Bu + B_d d, \]  
\[ y = Cx + Du + D_d d, \]  
where \( x \in \mathbb{R}^n \), \( u \in \mathbb{R}^{nu} \), \( d \in \mathbb{R}^{nd} \) and \( y \in \mathbb{R}^{ny} \). In this application the focus is on the plasma shape control loop and so a first loop for the control of the currents flowing in the PF coils is just considered as part of the plant (more details can be found in [1]). So, for our purposes, the control input \( u \) is represented by the 8 current references for the PF coils, while the controlled outputs \( y \) are represented by the \( n_p \) plasma shape parameters (gaps, X-point and strike points); typically \( n_p \geq 30 \). The disturbance vector \( d \) holds the poloidal beta \( \beta_p \), the internal inductance \( l_i \) and the plasma current \( I_p \). The plant is controlled by an a-priori given linear controller:

\[ \dot{x}_c = A_x x_c + B_e u_c + B_r r, \]  
\[ y_c = C_e x_c + D_e u_c + D_r r, \]

under the interconnection conditions:

\[ u_c = y, \]  
\[ u = y_c + y_a, \]

where \( x_c \in \mathbb{R}^{n}, u_c \in \mathbb{R}^{nr}, y_c \in \mathbb{R}^{ny} \) and \( r \in \mathbb{R}^{nr} \) is the (plasma shape) reference signal, i.e., the desired output.

In the considered case the a-priori given controller is the JET XSC. Assuming that the matrix transfer function \( P(s) = C(sI - A)^{-1}B + D \) from \( u \) to \( y \) has no pole at \( s = 0 \), let \( P^*: = P(0) \).

An input allocator block can be designed, described by the relations:

\[ w = -\rho \theta_0^{T} [I P^*] V J^T, \]  
\[ y_a = B_0 w, \]  
where \( B_0 \) is a suitable full column rank matrix, \( \rho > 0 \) specifies the convergence speed and \( f(u, e^*) \) is a suitable cost function (see [5] for more details) measuring the trade-off between the modified steady state value of the PF currents \( u^* \), and the associated plasma shape error \( e^* = r^* - y^* \). The CA is interconnected to the unconstrained closed-loop via the equations

\[ u_c = y - P^* y_a, \]  
\[ u = y_c + y_a, \]

as it can be seen in Fig. 1.

In particular, for this application the following cost function has been used (for brevity, the superscript \( \star \) indicating the steady-state is omitted):

\[ J(u, e) = \sum_{i=1}^{nu} a_i d(z(\bar{u}_i))^2 + \sum_{i=1}^{ny} b_i (e_i)^2, \]

where \( \bar{u}_i \) denotes the inputs normalized to the range \([-1, +1]\], \( d(z(\bar{u}_i)) = \text{sign}(\bar{u}_i)\text{max} \{0, |\bar{u}_i| - 1\} \) is the deadzone function, \( a_i \geq 0, i = 1, \ldots, n_u \) and \( b_i > 0, i = 1, \ldots, n_y \) are weight coefficients.

Such a function has the following features:

- penalizes separately each \( \bar{u}_i \) and \( e_i \);
- it does not penalize \( \bar{u}_i \) as long as \( |\bar{u}_i| \leq 1 \); and
- penalizes \( e_i \) quadratically (hence large values of \( e_i \) are penalized much more than small values);
- penalizes \( \bar{u}_i \) quadratically when \( |\bar{u}_i| \geq 1 \).

The above points imply that priority is given to keep the plasma shape errors \( e_i \)’s small, with relative weights specified by \( b_i \’s \), as long as the normalized currents \( \bar{u}_i \’s \) satisfy \( |\bar{u}_i| \leq 1 \); meanwhile, when the \( \bar{u}_i \’s \) are sufficiently outside the interval \([-1, +1]\), if the \( a_i \’s \) are sufficiently big with respect to the \( b_i \’s \), then priority is given to keep the \( \bar{u}_i \’s \) close to the interval \([-1, +1]\), even at the price of larger \( e_i \’s \). In such a way, normalizing the currents with respect to safety ranges smaller than the physical ones, the shape

1 The JET poloidal field coils system is made by 10 independent circuits. In particular, 8 of these circuits, namely P4T, IMB, SHP, PFX, D1, D2, D3 and D4, are used to control plasma shape, while the P1 circuit is used to control plasma current and FRFA is used to vertically stabilize the plasma column [6].
Fig. 3. Test Case A. PF currents when the CA is used to move both PFX and the SHP currents far from the saturation limits. Note that both the PFX and the SHP currents are kept equal their new upper bounds, in order to minimize error on the plasma shape tracking. It should also be noticed that the allocator slows down its reaction, once the PFX and SHP currents are in far from their limits.

Fig. 4. Test Case B. Plasma current variation. The plasma current is increased up to 4.5 MA.

tracking effort is automatically relaxed when the currents exceed these safety values.

From the point of view of convergence and stability, if the cost function in (7) is used, then, generalizing the results in [5], exponential convergence can be proven for sufficiently small values of the parameter $\rho$.

The stability of the whole control scheme, in fact, is based on the separation of the two time scales at which the allocator and the rest of the control loop work. About the optimality of the allocated steady state, the same arguments used in [5] can be exploited, noting that the cost function $J(u, e)$ is just a translation of the one proposed in [5] and so it is convex too.

3. Simulations

The problem of saturation avoidance for the shape control system at JET can be dealt with in the theoretical framework described in Section 2. The CA described in Section 2 has been integrated within the XSC and simulated with a linear model of the plasma. The following two test cases are presented in this section:

Case A – in this case the CA is adopted to move the PF currents far from their saturation limits, in order to operate a given scenario in a safer way;

Case B – in this case the CA allows to operate a given scenario with a larger plasma current without saturating the currents in the PF coils.

3.1. Test Case A

The JET pulse #78668 at $t = 13.4\text{ s}$ is considered in this test case. The equilibrium values of the plasma current, poloidal beta and internal inductance are $I_{\text{eq}} = 3.4\text{ MA}$, $\beta_{\text{eq}} = 0.29$, and $l_{\text{eq}} = 0.91$. In the considered configuration the currents in PFX and in the SHP circuits are very close to their limits. Indeed, their equilibrium values are equal to $I_{\text{PFX}} = 29.2\text{ kA}$ and $I_{\text{SHP}} = 32.7\text{ kA}$, while their upper bounds are equal to $I_{\text{PFX, max}} = 32.9\text{ kA}$ and $I_{\text{SHP, max}} = 38\text{ kA}$.

In this case the CA has been used to limit the currents in both the PFX and the SHP circuits. In particular, PFX can range in the interval $[4.9, 28.5]\text{ kA}$, while the current in SHP is limited to the range $[9.5, 28.5]\text{ kA}$. Furthermore, $I_{p}$ is kept constant and equal to $3.4\text{ MA}$. The plasma shape achieved at the steady–state is shown in Fig. 2, while the currents in the PF coils are reported in Fig. 3. In this simulation the plasma shape at $t = 13.4\text{ s}$ has been set as the reference for the XSC. The error in the outer upper zone can be reduced by increasing the PF limits up to 90% of the available operating space.

3.2. Test Case B

In this case we have considered the JET pulse #74177 at $t = 8.8\text{ s}$. For this equilibrium we have $I_{\text{eq}} = 4\text{ MA}$, $\beta_{\text{eq}} = 0.16$, and $l_{\text{eq}} = 0.75$. In this operative scenario plasma current is limited by the current in the D2 circuit, which is close to its saturation limit. Indeed, the equilibrium value of the current in the D2 circuit is equal to $I_{\text{D2, eq}} = 33.5\text{ kA}$, which is close to its lower saturation limit $(-37\text{ kA})$. The CA has been used to limit $I_{\text{D2}}$ in the range $[-31.45, -5.5]\text{ kA}$, and to increase the plasma current up to 4.5 MA (see Fig.
Fig. 5. Test Case B. Current in the PF circuits when $I_p$ is increased up to 4.5 MA and the CA is switched on. Note that the current in D2 is kept in the range $[-31.45, -5.5]$ kA.

Fig. 6. Test Case b. Plasma shape when $I_p$ is increased up to 4.5 MA. The shape reference is the dashed trace, while the dotted trace is the simulated shape.

4. Conclusions

A dynamic allocation technique has been proposed to handle the current saturations of the PF coils to improve the JET plasma shape control. All the analyses carried out have shown that the CA improves the performance of the XSC in terms of shape tracking and it allows to manage in the currents saturation in a flexible way.

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References