Universal vs tailored internal models in semiclassical robust output regulation for linear hybrid systems

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Summary. Output regulation for hybrid systems requires very complex input signals, whose generation in general cannot be based only on the knowledge of the exosystem (or its state, in the full information case); however, for a special class of hybrid systems semiclassical solutions are possible, such that the hybrid steady state motions required for output regulation can be generated by control inputs which are simple functions of the exosystem (or its state). Even in the semiclassical case, the complexity of the internal models required for robust output regulation motivates the search for simpler solutions, which is investigated in this paper.

An internal model based compensator for semiclassical robust output regulation

Following [1], trajectories of hybrid systems are parameterized by a double time variable \( (t, k) \) belonging to a hybrid time domain \( T \), which in the output regulation framework of [2] is a priori defined as \( T := \{ (t, k) : t \in [k\tau_M, (k + 1)\tau_M], k \in \mathbb{N} \} \), with \( \tau_M > 0 \) given (see Fig. 1a and also [3]). The time variable \( t \) measures the flow of continuous time, whereas the time variable \( k \) counts the number of jumps that the solution of the considered hybrid system has experienced.

Consider the following hybrid linear plant \( P \) and exosystem \( E \), where \( P \) has state \( x(t, k) \in \mathbb{R}^n \), control input \( u(t, k) \in \mathbb{R}^m \) and output \( e(t, k) \in \mathbb{R}^p \), and the state of the exosystem is \( w(t, k) \in \mathbb{R}^q \):

\[
\begin{align*}
\dot{x} &= Ax + Bu + Pw, & x^+ &= Ex + Rw, & e &= Cx + Qw, \\
\dot{w} &= Sw, & w^+ &= Jw,
\end{align*}
\]

which, due to the form of \( T \), must be understood in the sense that \( x(\cdot, \cdot) \) and \( w(\cdot, \cdot) \) satisfy the differential equations for almost all \( t \in (k\tau_M, (k + 1)\tau_M) \), and the difference equations (where \( x^+(t, k-1) := x(t, k) \)) when \( t = k\tau_M, k \in \mathbb{Z} \). The robustness aspect of the output regulation problem to be defined is related to the fact that \( P \) is only known to belong to a family \( F \), and to have a nominal description \( P^0 \in F \) (with data \( A^0, B^0, \ldots \)). Family \( F \) is characterized as follows.

Assumption 1 Each \( P \in F \) is described (possibly modulo a coordinate transformation) by matrices having the form

\[
A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ 0 & 0 & A_{33} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{21} & B_{22} & 0 \\ 0 & 0 & B_{32} \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & C_3 \end{bmatrix}, \quad P = \begin{bmatrix} P_1 & P_2 & P_3 \end{bmatrix}, \quad E = \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix}, \quad R = \begin{bmatrix} R_1 & R_2 & R_3 \end{bmatrix},
\]

where \( A_{11} \in \mathbb{R}^{n_1 \times n_1}, A_{22} \in \mathbb{R}^{n_2 \times n_2}, A_{33} \in \mathbb{R}^{n_3 \times n_3}, B_{11} \in \mathbb{R}^{n_1 \times m_1}, B_{32} \in \mathbb{R}^{n_3 \times m_2} \) with \( m_2 \geq p \).

Due to the constraint imposed by the fixed zero entries in (2) each plant in \( F \) can be uniquely identified with a vector of parameters \( f \in \mathbb{R}^z \), where \( z = [n(n + m + q) - (n_1 + m_1)(n_2 + n_3) - n_2n_3] + p(n_3 + q) + n(n + m + q) \), with \( f \) corresponding to \( P^0 \). Hence, an open neighborhood of \( P^0 \) in \( F \) can be identified with an open neighborhood of \( f \) in \( \mathbb{R}^z \). The structure in (2) is supposed to derive from the physical nature of the considered plant, see e.g. [4, 6, 7].

Problem 1 Given the nominal plant \( P^0 \in F \) and the exosystem \( E \), find, if possible, a robust output regulator using only measurements of \( e \) which achieves, for any \( P \in F_0 \) (where \( F_0 \subset F \) is an open neighborhood of \( P^0 \)), i) global exponential stability (GES) of the closed loop and ii) \( \lim_{t \to k \to +\infty} e(t, k) = 0 \) for all initial conditions (OR).

Under suitable hypotheses (which are the hybrid generalization of the classic hypotheses used for output regulation in the non hybrid context), the problem is solved by the robust regulator proposed in [7] and illustrated in Fig. 1b which is composed by a stabilizing unit \( K \) and an internal model unit \( I_M \) whose design is based on a hybrid internal model principle and which provides the internal model of the exosystem \( E \) which is necessary in order to achieve output regulation. In turn, the internal model unit \( I_M \) contains two subsystems (see Fig. 1c), a flow internal model \( I_F \) in charge of providing the inputs needed for regulation during flows, and a jump internal model \( I_J \) in charge of suitably resetting the regulator at

\[
\text{(a) The hybrid time domain } T.
\]

\[
\text{(b) The internal model based regulator.}
\]

\[
\text{(c) Detailed view of the internal model } I_M.
\]

Figure 1: Some key elements of the considered hybrid output regulation framework and its solution from [7].
each jump, taking into account the one-period dynamics of the exosystem given by \( w((k+1)\tau_M, k+1) = \hat{J}w(k\tau_M, k) \), where \( \hat{J} := J e^{St_M} \). It turns out that the state of \( \mathcal{I}_F \) has dimension \( n_F = p \cdot n_S \), due to the fact that for robust regulation \( \mathcal{I}_F \) must contain \( p \) copies of the essential flow dynamics of the exosystem, as given by the minimal polynomial \( \mu_S(s) \) of \( S \) having degree \( n_S \); in turn, the state of \( \mathcal{I}_J \) has dimension \( n_J = (n_3 + n_F) \cdot n_J \), due to the fact that in order to achieve the mentioned resetting (involving \( n_3 + n_F \) states) in a robust fashion, \( \mathcal{I}_J \) must contain \( n_3 + n_F \) copies of the essential one-period dynamics of the exosystem, as given by the minimal polynomial \( \mu_J(s) \) of \( J \) having degree \( n_J \).

**Tailored vs universal internal models**

The state of the overall internal model designed in \([7]\) has the remarkable dimension \( n_{IM} = n_J + n_F = (n_3 + pn_S)n_J + pn_S \). In order to get a more precise idea, consider the case in which \( S \) and \( J \) have all distinct eigenvalues, so that their minimal polynomial coincide with the respective characteristic polynomials and then have degrees \( n_S = n_J = q \). In such a case, \( n_{IM} = (n_3 + p(q + 1))q \); this number can be contrasted with the size of the internal model for the corresponding non-hybrid continuous-time case where \( \bar{w} = Sw \) (the discrete-time case \( w^+ = Jw \) is completely analogous), that is \( n_{IM, Nh} = pn_S = pq \), so that the difference amounts to \( n_{IM} - n_{IM, Nh} = (n_3 + pn_S)n_J = (n_3 + pq)q \).

It is worth to remark that, in the non-hybrid case, the same number \( n_{1M,NH} = pn_S \) is obtained independently from the fact that the available information consists in the state space description \( \bar{w} = Sw \) or in its minimal polynomial \( \mu_S(s) \). On the other hand, in the hybrid case the finer information about the state space description is actually crucial in order to design a more “tailored” internal model. Such a phenomenon is somewhat similar to what happens for stability, so that simple knowledge of the eigenvalues of \( S \) and \( J \) is not enough to determine stability of \( \mathcal{E} \), which can only be inferred by the eigenvalues of \( J \) (which can be computed by explicit knowledge of \( S \) and \( J \), but not from their eigenvalues alone).

The key phenomenon can be understood by considering that (the eigenvalues of) \( J \) capture the “character” of the exogenous signal over one “period” including a flow interval of length \( \tau_M \) and a single jump, whereas (the eigenvalues of) \( S \) describe the “shape” of such signals during the mentioned flow interval of length \( \tau_M \). Considering for brevity (in this extended abstract) only the case when all eigenvalues both of \( S \) and of \( J \) have multiplicity 1 in the respective minimal polynomial (that is, their Jordan blocks have dimension exactly 1, so that both matrices are diagonalizable), the above fact can be directly appreciated by writing down the free response of the exosystem \( \mathcal{E} \) in terms of its hybrid natural modes. Denoting by \( \{\lambda_1, \ldots, \lambda_{n_S}\} \) the eigenvalues of \( S \), by \( \{\delta_1, \ldots, \delta_{n_J}\} \) the eigenvalues of \( J \), and letting \( \tau := t - k\tau_M \) and \( w_0 := w(0,0) \), the free response of \( \mathcal{E} \) can be written as

\[
w(t, k) = e^{S\tau} \hat{J}^k w(0, 0) = e^{S\tau} \sum_{h=1}^{n_J} \delta_h^k w_{0,h} = \sum_{h=1}^{n_J} \sum_{\eta=1}^{n_S} e^{\lambda_h \tau} \delta_h^k w_{0,h,\eta} \tag{3}\]

where the \( w_{0,h}, h = 1, \ldots, n_J \), are a decomposition of \( w_0 \) with respect to the eigenspaces of \( J \), and \( w_{0,h,\eta}, h = 1, \ldots, n_J, \eta = 1, \ldots, n_S \) are a decomposition of \( w_{0,h} \) with respect to the eigenspaces of \( S \).

From the above expression, it is possible to see that if the only available information about \( \mathcal{E} \) is given by the minimal polynomials of \( S \) and \( J \), it is necessary to include in the flow internal model \( \mathcal{I}_F \) exactly \( p \) copies of the whole dynamics appearing in \([3]\) according to the recipe in \([7]\); on the other hand, if the exact values of \( S \) and \( J \) (hence of \( J \)) are known, it is possible to evaluate if all the terms in \([3]\) are present or not (this happens e.g. when, writing each eigenvector of \( J \) as a linear combination of the eigenvectors of \( S \), at least one eigenvector of \( S \) never appears).

It is worth noting that the above discussion applies to the flow internal model \( \mathcal{I}_F \), and then the jump internal model \( \mathcal{I}_J \) will always contain the same terms, determined only by the minimal polynomial of \( J \); however the number \( n_3 + n_F \) of copies of such dynamics included in \( \mathcal{I}_J \) may decrease due to the fact that a tailored design of \( \mathcal{I}_F \) has a state dimension \( n_F,\text{tailored} \) which is smaller than the state dimension \( n_F,\text{universal} \) for a universal design of \( \mathcal{I}_F \).

It is also important to stress that the increased dimension of a universal design yields the advantage of a more “robust” design, in the sense that it will work even in cases when the precise values of \( S \) and \( J \) are not accurately known or are subject to changes in time e.g. in some applications where some “leader” agents (modeled as exosystems) generate references for other “follower” agents, with interactions occurring according to a dynamical structure (a time varying graph, where links appear or disappear according to the distance or visibility between agents).

**References**


